

More Data Flow Analyses

Reading: NNH 2.1

17-654/17-765
Analysis of Software Artifacts
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Available Expressions

For each program point, which expressions *must* have already been computed, and not later modified, on all paths to the program point.

- Applications
 - Avoid re-computing expressions

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Available Expressions Example

$[x := a+b]^1;$	$AE_{exit}(1) = \{a+b\}$
$[y := a*b]^2;$	$AE_{exit}(2) = \{a+b, a*b\}$
while $[y > a+b]^3$ do	$AE_{exit}(3) = \{a+b\}$
$[a := a+1]^4;$	$AE_{exit}(4) = \emptyset$
$[x := a+b]^5$	$AE_{exit}(5) = \{a+b\}$

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Available Expressions Equations

$[x := a+b]^1;$	$AE_{entry}(1) = \emptyset$
$[y := a*b]^2;$	$AE_{entry}(2) = AE_{exit}(1)$
while $[y > a+b]^3$ do	$AE_{entry}(3) = AE_{exit}(2) \cap AE_{exit}(5)$
$[a := a+1]^4;$	$AE_{entry}(4) = AE_{exit}(3)$
$[x := a+b]^5$	$AE_{entry}(5) = AE_{exit}(4)$
	$AE_{exit}(1) = AE_{entry}(1) \cup \{a+b\}$
	$AE_{exit}(2) = AE_{entry}(2) \cup \{a*b\}$
	$AE_{exit}(3) = AE_{entry}(3) \cup \{a+b\}$
	$AE_{exit}(4) = AE_{entry}(4) \setminus \{a+b, a*b, a+1\}$
	$AE_{exit}(5) = AE_{entry}(5) \cup \{a+b\}$

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Another Example

Program	Dataflow Eqns	Solutions
$[x := a+b]^1;$	$AE_{entry}(1) = \emptyset$	\emptyset
while $[true]^2$ do	$AE_{entry}(2) = AE_{exit}(1) \cap AE_{exit}(3)$	\emptyset or $\{a+b\}$
$[skip]^3;$	$AE_{entry}(3) = AE_{exit}(2)$	\emptyset or $\{a+b\}$
	$AE_{exit}(1) = AE_{entry}(1) \cup \{a+b\}$	$\{a+b\}$
	$AE_{exit}(2) = AE_{entry}(2)$	\emptyset or $\{a+b\}$
	$AE_{exit}(3) = AE_{entry}(3)$	\emptyset or $\{a+b\}$

What's the fixed point of these equations?
– Which is most informative?

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Available Exp. vs. Reaching Defs.

- Available Exp. **Must analysis**
 - Initial dataflow values: *empty* sets
 - *Intersection* at control flow merge
 - Precision: want *greatest* fixed point
 - Safety: err on the side of *smaller* sets
- Reaching Defs. **May analysis**
 - Initial dataflow values: *universal* sets
 - *Union* at control flow merge
 - Precision: want *least* fixed point
 - Safety: err on the side of *larger* sets

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Kill and Gen Sets

- Useful for defining transfer functions
 - Kill: dataflow facts the statement removes
 - Gen: dataflow facts the statement adds
 - Kill always comes first

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Kill and Gen for Available Expressions

$$\text{kill}_{\text{AE}}([x := a]^{\ell}) = \{a' \in \mathbf{AExp}_* \mid x \in \text{FV}(a')\}$$

$$\text{kill}_{\text{AE}}([\text{skip}]^{\ell}) = \emptyset$$

$$\text{kill}_{\text{AE}}([b]^{\ell}) = \emptyset$$

$$\text{gen}_{\text{AE}}([x := a]^{\ell}) = \{a' \in \mathbf{AExp}(a) \mid x \notin \text{FV}(a')\}$$

$$\text{gen}_{\text{AE}}([\text{skip}]^{\ell}) = \emptyset$$

$$\text{gen}_{\text{AE}}([b]^{\ell}) = \mathbf{AExp}(b)$$

We should define this!

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Definition of $\mathbf{AExp}(a)$, $\mathbf{AExp}(b)$

$$\mathbf{AExp}(x) = \emptyset$$

$$\mathbf{AExp}(n) = \emptyset$$

$$\mathbf{AExp}(a_1 \text{ op}_a a_2) = \{a_1 \text{ op}_a a_2\} \cup \mathbf{AExp}(a_1) \cup \mathbf{AExp}(a_2)$$

$$\mathbf{AExp}(\text{true}) = \emptyset$$

$$\mathbf{AExp}(\text{false}) = \emptyset$$

$$\mathbf{AExp}(\text{not } b) = \{\text{not } b\} \cup \mathbf{AExp}(b)$$

$$\mathbf{AExp}(b_1 \text{ op}_b b_2) = \{b_1 \text{ op}_b b_2\} \cup \mathbf{AExp}(b_1) \cup \mathbf{AExp}(b_2)$$

$$\mathbf{AExp}(a_1 \text{ op}_a a_2) = \{a_1 \text{ op}_a a_2\} \cup \mathbf{AExp}(a_1) \cup \mathbf{AExp}(a_2)$$

You'll do something similar on the homework

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Kill and Gen for Reaching Definitions

$$\text{kill}_{\text{RD}}([x := a]^{\ell}) = \{(x, ?)\} \cup \{(x, \ell') \mid \ell' \text{ is any label}\}$$

$$\text{kill}_{\text{RD}}([\text{skip}]^{\ell}) = \emptyset$$

$$\text{kill}_{\text{RD}}([b]^{\ell}) = \emptyset$$

$$\text{gen}_{\text{RD}}([x := a]^{\ell}) = \{(x, \ell)\}$$

$$\text{gen}_{\text{RD}}([\text{skip}]^{\ell}) = \emptyset$$

$$\text{gen}_{\text{RD}}([b]^{\ell}) = \emptyset$$

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Some Notation

- This will help us describe analyses in a more precise and general way

- $\text{init}(S)$ – the label of the first statement in S
- $\text{final}(S)$ – the set of labels of the last statements in S
 - the last statement on each branch of an if
 - the test of a while
- $\text{blocks}(S)$ – the set of primitive statements and tests in S
- $\text{labels}(S)$ – the set of labels of blocks in S
- $\text{flow}(S)$ – $\{(\ell, \ell') \mid \text{control may transfer from block } \ell \text{ to block } \ell'\}$
 - A pair for each edge in the control flow graph

- The text defines these formally

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General Data Flow Equations

Available Expressions

$$\text{AE}_{\text{entry}}(\ell) = \begin{cases} \emptyset & \text{if } (\ell = \text{init}(S)) \\ \cap \{ \text{AE}_{\text{exit}}(\ell') \mid (\ell', \ell) \in \text{flow}(S) \} & \text{otherwise} \end{cases}$$

$$\text{AE}_{\text{exit}}(\ell) = (\text{AE}_{\text{entry}}(\ell) \setminus \text{kill}_{\text{AE}}(B^{\ell})) \cup \text{gen}_{\text{AE}}(B^{\ell})$$

Reaching Definitions

$$\text{RD}_{\text{entry}}(\ell) = \begin{cases} \{(x, ?) \mid x \in \text{FV}(S)\} & \text{if } \ell = \text{init}(S) \\ \cup \{ \text{RD}_{\text{exit}}(\ell') \mid (\ell', \ell) \in \text{flow}(S) \} & \text{otherwise} \end{cases}$$

$$\text{RD}_{\text{exit}}(\ell) = (\text{RD}_{\text{entry}}(\ell) \setminus \text{kill}_{\text{RD}}(B^{\ell})) \cup \text{gen}_{\text{RD}}(B^{\ell})$$

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Constant Propagation

- For each program point, what value *may* each variable hold?

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Constant Propagation

$$CP_{\text{entry}}(\ell) = \{(x,n) \mid x \in FV(S.), n \in \mathbb{N}\} \quad \text{if } (\ell = \text{init}(S.)) \\ = \cup \{ CP_{\text{exit}}(\ell') \mid (\ell', \ell) \in \text{flow}(S.) \} \quad \text{otherwise}$$

$$CP_{\text{exit}}(\ell) = (CP_{\text{entry}}(\ell) \setminus \text{kill}_{CP}(B^{\ell})) \cup \text{gen}_{CP}(B^{\ell})$$

$$\text{kill}_{CP}([x := a]^{\ell}) = \{(x,n) \mid n \in \mathbb{N}\}$$

$$\text{kill}_{CP}([\text{skip}]^{\ell}) = \emptyset$$

$$\text{kill}_{CP}([b]^{\ell}) = \emptyset$$

$$\text{gen}_{CP}([x := a]^{\ell}) = \{(x, n) \mid n \in CP^{\ell}(a)\}$$

$$\text{gen}_{CP}([\text{skip}]^{\ell}) = \emptyset$$

$$\text{gen}_{CP}([b]^{\ell}) = \emptyset$$

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Constant Propagation

$$CP^{\ell}(x) = \{ CP_{\text{entry}}(\ell)(x) \}$$

$$CP^{\ell}(n) = \{ n \}$$

$$CP^{\ell}(a_1 \text{ op}_a a_2) = CP^{\ell}(a_1) \widehat{\text{op}}_a CP^{\ell}(a_2)$$

$$\text{set}_1 \widehat{\text{op}}_a \text{set}_2 = \{ n_1 \text{ op}_a n_2 \mid n_1 \in \text{set}_1, n_2 \in \text{set}_2 \}$$

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Constant Propagation Example

[y := 5] ¹ ;		x	y	z	w
[z := 1+y] ² ;	AE _{exit} (1) =	?	5	?	?
if [x = 5] ³	AE _{exit} (2) =	?	5	6	?
then [w := x+1] ⁴ ;	AE _{exit} (3) =	?	5	6	?
else [w := y+1] ⁵	AE _{exit} (4) =	?	5	6	?
[skip] ⁶	AE _{exit} (5) =	?	5	6	6
	AE _{exit} (6) =	?	5	6	?

Here ? is a shorthand for the set of all integers

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Constant Propagation Example

[y := 5] ¹ ;		x	y	z	w
[z := 1+y] ² ;	AE _{exit} (1) =	?	5	?	?
if [x = 5] ³	AE _{exit} (2) =	?	5	6	?
then [w := x+1] ⁴ ;	AE _{exit} (3) =	?	5	6	?
else [w := y+1] ⁵	AE _{exit} (4) =	?	5	6	?
[skip] ⁶	AE _{exit} (5) =	?	5	6	6
	AE _{exit} (6) =	?	5	6	?

But we know that x=5 at statement 4. Can we do better?

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Constant Propagation, Take 2

$$CP_{\text{entry}}(\ell) = \{(x,n) \mid x \in FV(S.), n \in \mathbb{N}\} \quad \text{if } (\ell = \text{init}(S.)) \\ = \cup \{ CP_{\text{exit}}(\ell') \mid (\ell', \ell) \in \text{flow}(S.) \} \quad \text{otherwise}$$

$$CP_{\text{I}}(\ell) = (CP_{\text{entry}}(\ell) \setminus \text{kill}_{CP,I}(B^{\ell})) \cup \text{gen}_{CP,I}(B^{\ell}) \\ CP_{\text{F}}(\ell) = (CP_{\text{entry}}(\ell) \setminus \text{kill}_{CP,F}(B^{\ell})) \cup \text{gen}_{CP,F}(B^{\ell})$$

$$\text{kill}_{CP,I}([x := a]^{\ell}) = \{(x, n) \mid n \in \mathbb{N}\}$$

$$\text{kill}_{CP,I}([\text{skip}]^{\ell}) = \emptyset$$

$$\text{kill}_{CP,I}([b]^{\ell}) = \{(x, n) \mid n \in \mathbb{N} \text{ and } n \neq m\}$$

$$= \emptyset$$

$$\text{kill}_{CP,F}([b]^{\ell}) = \{(x, m)\}$$

$$= \emptyset$$

when $b = (x=a)$ and $CP^{\ell}(a)=(m)$

otherwise

when $b = (x=a)$ and $CP^{\ell}(a)=(m)$

otherwise

$$\text{gen}_{CP,I}([x := a]^{\ell}) = \{(x, n) \mid n \in CP^{\ell}(a)\}$$

$$\text{gen}_{CP,I}([\text{skip}]^{\ell}) = \emptyset$$

$$\text{gen}_{CP,I}([b]^{\ell}) = \emptyset$$

$$\text{gen}_{CP,F}([b]^{\ell}) = \emptyset$$

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Constant Propagation Example

<code>[y := 5]¹;</code>		<u>x</u>	<u>y</u>	<u>z</u>	<u>w</u>
<code>[z := 1+y]²;</code>	$AE_{\text{exit}}(1) =$?	5	?	?
<code>if [x = 5]³</code>	$AE_{\text{exit}}(2) =$?	5	6	?
<code>then [w := x+1]⁴;</code>	$AE_T(3) =$	5	5	6	?
<code>else [w := y+1]⁵</code>	$AE_F(3) =$? \ 5	5	6	?
<code>[skip]⁶</code>	$AE_{\text{exit}}(4) =$	5	5	6	6
	$AE_{\text{exit}}(5) =$? \ 5	5	6	6
	$AE_{\text{exit}}(6) =$?	5	6	6

Keeping track of data flow values separately on each branch supports a more precise final result.

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